

KALEIDOSCOPE-ROULETTE: THE RESONANCE PHENOMENA IN PERCEPTION GAMES

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math.OC/9905180

Kaleidoscope-roulettes, a proper class of perception games, is described. Kaleidoscope-roulette is defined as a perception and, hence, verbalizable interactive game, whose hidden dialogue consists of quasirandom sequences of “words”. The resonance phenomena in such games and their controlling are discussed.

The mathematical formalism of interactive games, which extends one of ordinary games [1] and is based on the concept of an interactive control, was recently proposed by the author [2] to take into account the complex composition of controls of a real human person, which are often complicated couplings of his/her cognitive and known controls with the unknown subconscious behavioral reactions. This formalism is applicable also to the description of external unknown influences and, thus, is useful for problems in computer science (e.g. the semi-artificially controlled distribution of resources), mathematical economics (e.g. the financial games with unknown dynamical factors) and sociology (e.g. the collective decision making).

Recently, two proper classes of the interactive games were introduced: the verbalizable interactive games [3] and the perception games [4]. The first class appeared as a result of the interactive game theoretical description of dialogues as psycholinguistic phenomena and the second one was obtained as such description of perception processes and the image understanding. Each perception game is a verbalizable interactive game.

This article is devoted to a new proper subclass of interactive games, namely, to the kaleidoscope-roulettes. Kaleidoscope-roulette is defined as a perception and, hence, verbalizable interactive game, whose hidden dialogue consists of quasirandom sequences of “words”. The resonance phenomena in such games are investigated.

Though kaleidoscope-roulettes are naturally associated with an entertainment their real applications may be far beyond it due to their origin.

1991 *Mathematics Subject Classification.* 90D20 (Primary) 90D80, 49N55, 93C41, 93B52 (Secondary).

Key words and phrases. Interactive game, Perception game, Kaleidoscope-roulette, Resonance.

I. INTERACTIVE GAMES

1.1. Interactive systems and intention fields.

Definition 1 [2]. An *interactive system* (with n *interactive controls*) is a control system with n independent controls coupled with unknown or incompletely known feedbacks (the feedbacks as well as their couplings with controls are of a so complicated nature that their can not be described completely). An *interactive game* is a game with interactive controls of each player.

Below we shall consider only deterministic and differential interactive systems. In this case the general interactive system may be written in the form:

$$(1) \quad \dot{\varphi} = \Phi(\varphi, u_1, u_2, \dots, u_n),$$

where φ characterizes the state of the system and u_i are the interactive controls:

$$u_i(t) = u_i(u_i^\circ(t), [\varphi(\tau)]|_{\tau \leq t}),$$

i.e. the independent controls $u_i^\circ(t)$ coupled with the feedbacks on $[\varphi(\tau)]|_{\tau \leq t}$. One may suppose that the feedbacks are integrodifferential on t .

Proposition [2]. *Each interactive system (1) may be transformed to the form (2) below (which is not, however, unique):*

$$(2) \quad \dot{\varphi} = \tilde{\Phi}(\varphi, \xi),$$

where the magnitude ξ (with infinite degrees of freedom as a rule) obeys the equation

$$(3) \quad \dot{\xi} = \Xi(\xi, \varphi, \tilde{u}_1, \tilde{u}_2, \dots, \tilde{u}_n),$$

where \tilde{u}_i are the interactive controls of the form $\tilde{u}_i(t) = \tilde{u}_i(u_i^\circ(t); \varphi(t), \xi(t))$ (here the dependence of \tilde{u}_i on $\xi(t)$ and $\varphi(t)$ is differential on t , i.e. the feedbacks are precisely of the form $\tilde{u}_i(t) = \tilde{u}_i(u_i^\circ(t); \varphi(t), \xi(t), \dot{\varphi}(t), \dot{\xi}(t), \ddot{\varphi}(t), \ddot{\xi}(t), \dots, \varphi^{(k)}(t), \xi^{(k)}(t))$).

Remark 1. One may exclude $\varphi(t)$ from the feedbacks in the interactive controls $\tilde{u}_i(t)$. One may also exclude the derivatives of ξ and φ on t from the feedbacks.

Definition 2 [2]. The magnitude ξ with its dynamical equations (3) and its contribution into the interactive controls \tilde{u}_i will be called the *intention field*.

Note that the theorem holds true for the interactive games. In practice, the intention fields may be often considered as a field-theoretic description of subconscious individual and collective behavioral reactions. However, they may be used also the accounting of unknown or incompletely known external influences. Therefore, such approach is applicable to problems of computer science (e.g. semi-automatically controlled resource distribution) or mathematical economics (e.g. financial games with unknown factors). The interactive games with the differential dependence of feedbacks are called *differential*. Thus, the theorem states a possibility of a reduction of any interactive game to a differential interactive game by introduction of additional parameters – *the intention fields*.

1.2. Some generalizations. The interactive games introduced above may be generalized in the following ways.

The first way, which leads to the *indeterminate interactive games*, is based on the idea that the pure controls $u_i^\circ(t)$ and the interactive controls $u_i(t)$ should not be obligatory related in the considered way. More generally one should only postulate that there are some time-independent quantities $F_\alpha(u_i(t), u_i^\circ(t), \varphi(t), \dots, \varphi^{(k)}(t))$ for the independent magnitudes $u_i(t)$ and $u_i^\circ(t)$. Such claim is evidently weaker than one of Def.1. For instance, one may consider the inverse dependence of the pure and interactive controls: $u_i^\circ(t) = u_i^\circ(u_i(t), \varphi(t), \dots, \varphi^{(k)}(t))$.

The inverse dependence of the pure and interactive controls has a nice psychological interpretation. Instead of thinking of our action consisting of the conscious and unconscious parts and interpreting the least as unknown feedbacks, which “dress” the first, one is able to consider our action as a single whole whereas the act of consciousness is in the extraction of a part, which it declares as its property.

The second way, which leads to the *coalition interactive games*, is based on the idea to consider the games with coalitions of actions and to claim that the interactive controls belong to such coalitions. In this case the evolution equations have the form

$$\dot{\varphi} = \Phi(\varphi, v_1, \dots, v_m),$$

where v_i is the interactive control of the i -th coalition. If the i -th coalition is defined by the subset I_i of all players then

$$v_i = v_i(\varphi(t), \dots, \varphi^{(k)}(t), u_j^\circ | j \in I_i).$$

Certainly, the intersections of different sets I_i may be non-empty so that any player may belong to several coalitions of actions. Def.1 gives the particular case when $I_i = \{i\}$.

The coalition interactive games may be an effective tool for an analysis of the collective decision making in the real coalition games that spread the applicability of the elaborating interactive game theory to the diverse problems of sociology.

1.3. Differential interactive games and their ε -representations.

Definition 3 [3]. The ε -representation of differential interactive game is a representation of the differential feedbacks in the form

$$(4) \quad u_i(t) = u_i(u_i^\circ, \varphi(t), \dots, \varphi^{(k)}(t); \varepsilon_i(t))$$

with the *known* function u_i of all its arguments, where the magnitudes $\varepsilon_i(t) \in \mathcal{E}$ are *unknown* functions of u_i° and $\varphi(t)$ with its higher derivatives:

$$\varepsilon_i(t) = \varepsilon_i(u_i^\circ(t), \varphi(t), \dot{\varphi}(t), \dots, \varphi^{(k)}(t)).$$

It is interesting to consider several different ε -representations simultaneously. For such simultaneous ε -representations with ε -parameters $\varepsilon_i^{(\alpha)}$ a crucial role is played by the time-independent relations between them:

$$F_\beta(\varepsilon_i^{(1)}, \dots, \varepsilon_i^{(\alpha)}, \dots, \varepsilon_i^{(N)}; u_i^\circ, \varphi, \dots, \varphi^{(k)}) \equiv 0,$$

which are called the *correlation integrals*. Certainly, in practice the correlation integrals are determined *a posteriori* and, thus they contain an important information on the interactive game. Using the sufficient number of correlation integrals one is able to construct various algebraic structures in analogy to the correlation functions in statistical physics and quantum field theory.

II. DIALOGUES AND THE VERBALIZABLE INTERACTIVE GAMES. PERCEPTION GAMES.

2.1. Dialogues as interactive games. The verbalization.

Dialogues as psycholinguistic phenomena can be formalized in terms of interactive games. First of all, note that one is able to consider interactive games of discrete time as well as interactive games of continuous time above.

Defintion 4A (the naïve definition of dialogues) [3]. The *dialogue* is a 2-person interactive game of discrete time with intention fields of continuous time.

The states and the controls of a dialogue correspond to the speech whereas the intention fields describe the understanding.

Let us give the formal mathematical definition of dialogues now.

Definition 4B (the formal definition of dialogues) [3]. The *dialogue* is a 2-person interactive game of discrete time of the form

$$(5) \quad \varphi_n = \Phi(\varphi_{n-1}, \vec{v}_n, \xi(\tau) | t_{n-1} \leq \tau \leq t_n).$$

Here $\varphi_n = \varphi(t_n)$ are the states of the system at the moments t_n ($t_0 < t_1 < t_2 < \dots < t_n < \dots$), $\vec{v}_n = \vec{v}(t_n) = (v_1(t_n), v_2(t_n))$ are the interactive controls at the same moments; $\xi(\tau)$ are the intention fields of continuous time with evolution equations

$$(6) \quad \dot{\xi}(t) = \Xi(\xi(t), \vec{u}(t)),$$

where $\vec{u}(t) = (u_1(t), u_2(t))$ are continuous interactive controls with ε -represented couplings of feedbacks:

$$u_i(t) = u_i(u_i^\circ(t), \xi(t); \varepsilon_i(t)).$$

The states φ_n and the interactive controls \vec{v}_n are certain *known* functions of the form

$$(7) \quad \begin{aligned} \varphi_n &= \varphi_n(\vec{\varepsilon}(\tau), \xi(\tau) | t_{n-1} \leq \tau \leq t_n), \\ \vec{v}_n &= \vec{v}_n(\vec{u}^\circ(\tau), \xi(\tau) | t_{n-1} \leq \tau \leq t_n). \end{aligned}$$

Note that the most nontrivial part of mathematical formalization of dialogues is the claim that the states of the dialogue (which describe a speech) are certain “mean values” of the ε -parameters of the intention fields (which describe the understanding).

Important. The definition of dialogue may be generalized on arbitrary number of players and below we shall consider any number n of them, e.g. $n = 1$ or $n = 3$, though it slightly contradicts to the common meaning of the word “dialogue”.

An embedding of dialogues into the interactive game theoretical picture generates the reciprocal problem: how to interpret an arbitrary differential interactive game as a dialogue. Such interpretation will be called the *verbalization*.

Definition 5 [3]. A differential interactive game of the form

$$\dot{\varphi}(t) = \Phi(\varphi(t), \vec{u}(t))$$

with ε -represented couplings of feedbacks

$$u_i(t) = u_i(u_i^\circ(t), \varphi(t), \dot{\varphi}(t), \ddot{\varphi}(t), \dots, \varphi^{(k)}(t); \varepsilon_i(t))$$

is called *verbalizable* if there exist a *a posteriori* partition $t_0 < t_1 < t_2 < \dots < t_n < \dots$ and the integrodifferential functionals

$$(8) \quad \begin{aligned} \omega_n(\vec{\varepsilon}(\tau), \varphi(\tau) | t_{n-1} \leq \tau \leq t_n), \\ \vec{v}_n(\vec{u}^\circ(\tau), \varphi(\tau) | t_{n-1} \leq \tau \leq t_n) \end{aligned}$$

such that

$$(9) \quad \omega_n = \Omega(\omega_{n-1}, v_n; \varphi(\tau) | t_{n-1} \leq \tau \leq t_n).$$

The verbalizable differential interactive games realize a dialogue in sense of Def.4.

Remark 2. One may include ω_n explicitly into the evolution equations for φ

$$\dot{\varphi}(\tau) = \Phi(\varphi(\tau), \vec{u}(\tau); \omega_n), \quad \tau \in [t_n, t_{n+1}].$$

as well as into the feedbacks and their couplings.

The main heuristic hypothesis is that all differential interactive games “which appear in practice” are verbalizable. The verbalization means that the states of a differential interactive game are interpreted as intention fields of a hidden dialogue and the problem is to describe such dialogue completely. If a differential interactive game is verbalizable one is able to consider many linguistic (e.g. the formal grammar of a related hidden dialogue) or psycholinguistic (e.g. the dynamical correlation of various implications) aspects of it.

During the verbalization it is a problem to determine the moments t_i . A way to the solution lies in the structure of ε -representation. Let the space E of all admissible values of ε -parameters be a CW-complex. Then t_i are just the moments of transition of the ε -parameters to a new cell.

2.2. Perception games.

Definition 6. The *perception game* is a multistage verbalizable game (no matter finite or infinite) for which the intervals $[t_i, t_{i+1}]$ are just the sets. The conditions of their finishing depends only on the current value of φ and the state of ω at the beginning of the set. The initial position of the set is the final position of the preceeding one.

Practically, the definition describes the discrete character of the perception and the image understanding. For example, the goal of a concrete set may be to perceive or to understand certain detail of the whole image. Another example is a continuous perception of the moving or changing object.

Note that the definition of perception games is applicable to various forms of perception, though the most interesting one is the visual perception. The proposed definition allows to take into account the dialogical character of the image understanding and to consider the visual perception, image understanding and the verbal (and nonverbal) dialogues together. It may be extremely useful for the analysis of collective perception, understanding and controlling processes in the dynamical environments – sports, dancings, martial arts, the collective controlling of moving objects, etc. On the other hand this definition explicates the self-organizing features of human perception, which may be unraveled by the game theoretical analysis. And, finally, the definition put a basis for a systematical application of the linguistic (e.g. formal grammars) and psycholinguistic methods to the image understanding as a verbalizable interactive game with a mathematical rigor.

III. KALEIDOSCOPE-ROULETTES AND THE RESONANCE PHENOMENA

3.1. Kaleidoscope-roulette. The kaleidoscope-roulette is a result of the attempt to combine the kaleidoscope, one of the simplest and effective visual game, with the roulette essentially using the elements of randomness and the treatment of resonances. The main idea is to substitute random sequences of roulette by the quasirandom sequences, which may be generated by the interactive kaleidoscope. The obtained formal definition is below.

Definition 7. *Kaleidoscope-roulette* is a perception game with a quasirandom sequence of quantites $\{\omega_n\}$.

Certainly, the explicit form of functionals (8) is not known to the players.

Many concrete versions of kaleidoscope-roulettes are constructed. Though they are naturally associated with an entertainment their real applications may be far beyond it due to their origin and the abstract character of their definition.

3.2. The resonance phenomena in kaleidoscope-roulettes. Though the sequence $\{\omega_n\}$ is quasirandom the equations (9) for them may have the resonance solutions. The resonance means a dynamical correlation of two quasirandom sequences $\{v_n\}$ and $\{\omega_n\}$ whatever φ is realized. In such case the quantities $\{v_n\}$ may be comprehended as “fortune”, what is not senseless in contrast to the ordinary roulette. However, v_n are *interactive* controls and their explicit dependence on \vec{u}° and φ is not known. Nevertheless, one is able to use *a posteriori* analysis and short-term predictions based on it (cf.[5]) if the time interval Δt in the short-term predictions is not less than the interval $t_{n+1} - t_n$. To do it one should slightly improve constructions of [5] to take the discrete-time character of v_n into account. It allows to perform the short-term controlling of the resonances in a kaleidoscope-roulette if they are observed. The conditions of applicability of short-time predictions to the controlling of resonances may be expressed in the following form: one should claim that *variations of the interactivity should be slower than the change of sets in the considered multistage game*.

Remark 3. The possibility to control resonances by v_n using its short-term predictions does not contradict to its quasirandomness, because v_n is quasirandom with respect to v_{n-1} but not to $\varphi(\tau)$ ($\tau \in [t_n, t_{n+1}]$).

IV. CONCLUSIONS

Kaleidoscope-roulettes, a proper class of perception games, is described. They are defined as perception and, hence, verbalizable interactive games, whose hidden dialogue consists of quasirandom sequences of “words”. The resonance phenomena in such games and their controlling are discussed. A possibility of the short-term controlling of resonances in the kaleidoscope-roulettes is doubtless an intriguing feature for its use for the entertainment purposes as well as far beyond them.

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